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Abstracts

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The p-adic methods in the design of physically unclonable functions

Vladimir Anashin

Physically unclonable functions (PUFs) are currently attracting great interest in information security. In [1] the PUFs are described as follows:

> A physically unclonable function or PUF is best described as "an expression of an inherent and unclonable instance-specific feature of a physical object". This means that through physical reasoning it is shown that producing a physical clone of a PUF is extremely hard or impossible. The physical motivation for claiming unclonability of an inherent instance-specific feature is found in the technical limitations of the production of physical objects.

There are various PUF constructions based on various physical principles and effects, see [1]. In the talk, two types of PUFs will be concerned:

- superlattice-based PUFs (SL-PUFs, for short), and
- field-programmable gate array-based PUFs (FPGA-PUFs)

The core of SL-PUFs are semiconductor superlattices which exhibit chaotic behaviour [2, 3] while FPGA-PUFs are programmable chips whose internal logic cannot be cloned after programming. The idea of using T-functions to construct FPGA-PUFs was first stated in [4].

In the talk it will be shown that functioning of both PUFs can be mathematically described in terms of 1-Lipschitz maps of 2-adic integers (also known as T-functions).

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A stochastic p-adic model of the capillary flow in random medium

Oleksandra Antoniouk

We develop the p-adic model of propagation of fluids in capillary networks in a porous random medium. The hierarchic structure of a system of capillaries is mathematically modeled by endowing trees of capillaries with the structure of an ultrametric space. Considerations are restricted to the case of idealized networks represented by homogeneous p-trees with p branches leaving each vertex, where $p > 1$ is a prime number. Such trees are realized as the fields of p-adic numbers. We introduce and study an inhomogeneous Markov process describing the penetration of fluid into a porous random medium.

Application of p-Adic Analysis Methods in Describing Markov Processes on Ultrametric Spaces Isometrically Embedded into Q_p

Albert Bikulov

We propose a method for describing stationary Markov processes on the class of ultrametric spaces U isometrically embedded in the field Q_p of p-adic numbers. This method is capable of reducing the study of such processes to the investigation of processes on Q_p. Thereby the traditional machinery of p-adic mathematical physics can be applied to calculate the characteristics of stationary Markov processes on such spaces. The Cauchy problem for the Kolmogorov-Feller equation of a stationary Markov process on such spaces is shown as being reducible to the Cauchy problem for a pseudo-differential equation on Q_p with non-translation-invariant measure $m(x)dx$. The spectrum of the pseudo-differential operator of the Kolmogorov-Feller equation on with measure $m(x)dx$ is found. Orthonormal basis of real valued functions for $L^2(Q_p, m(x)dx)$ is constructed from the eigenfunctions of this operator.

Generalised diffusion on the moduli space of p-adic Mumford curves

Patrick Erik Bradley

In this talk a construction of pseudo-differential operators of Taibleson-Vladimirov type on the moduli space of Mumford curves of genus g over non-archimedean local fields is given by using the Gerritzen-Herrlich Teichmüller space of discrete representations of finitely generated free groups in g generators as projective linear transformations. Corresponding Cauchy problems are stated and solved.

Non-singular solutions in ghost-free higher derivative gravity

Luca Buoninfante

One of the most straightforward attempt aimed to complete Einstein's general relativity in the ultraviolet (or short-distance) regime was to introduce quadratic curvature terms in the gravitational action besides the Einstein-Hilbert term, as for example $\mathcal{R}^2$ and $\mathcal{R}_{\mu\nu}\mathcal{R}^{\mu\nu}$. Such an action turns out to be power counting renormalizable, but suffers from the presence of a massive spin-2 ghost degree of freedom, which causes classical Hamiltonian instabilities and breaks the unitarity condition at the quantum level [1].

Recently, it has been pointed out that a possible way to ameliorate the issue of ghost is to go beyond finite order derivative theories, and to modify the Einstein-Hilbert action by introducing differential operators made up of *infinite* order covariant derivatives, thus giving up the locality principle. In fact, by generalizing the Einstein-Hilbert action with quadratic curvature terms made up of nonlocal (i.e. non-polynomial) operators, one can formulate a quantum theory of gravity which is unitary and that shows an improved ultraviolet behaviour [2]. The nonlocal differential operators are required to be made up of *exponential of entire functions* in order to avoid the presence of ghost-like degrees of freedom in the graviton propagator and preserve the unitarity condition [2].

In this talk, after briefly reviewing the main aspects of nonlocal (infinite derivative) field theories [5], we will focus on spherically symmetric spacetime solutions of nonlocal gravitational theories. We will find a linearized metric solution for a (neutral and charged) point-like source, and show that it is nonsingular [3]. By analysing all the curvature tensors one can capture and understand the physical implications due to the nonlocal nature of the gravitational interaction. In particular, the Kretschmann invariant turns out to be non-singular, while all the Weyl tensor components vanish at the origin meaning that the metric tends to be conformally-flat at $r = 0$. Similar features can be also found in the case of a Delta Dirac distribution on a ring for which no Kerr-like singularity appears [4].

Finally, we will discuss and make a comparison among different nonlocal operators which satisfy the ghost-free condition [6].

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A space of p-adic test functions

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We develop the p-adic model of propagation of fluids in capillary networks in a porous random medium. The hierarchic structure of a system of capillaries is mathematically modeled by endowing trees of capillaries with the structure of an ultrametric space. Considerations are restricted to the case of idealized networks represented by homogeneous p-trees with p branches leaving each vertex, where $p > 1$ is a prime number. Such trees are realized as the fields of p-adic numbers. We introduce and study an inhomogeneous Markov process describing the penetration of fluid into a porous random medium.

Cosmological Solutions of a New Nonlocal Gravity Model

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Despite of numerous significant phenomenological confirmations and many nice theoretical properties, General Relativity (GR) is not final theory of gravity. Problems mainly come from quantum gravity, cosmology and astrophysics. For example, if GR is applicable to the universe as a whole and the universe is homogeneous and isotropic, then it follows that the universe contains about 68% of dark energy, 27% of dark matter and only about 5% of visible matter. However, validity of GR at the very large cosmic scale is not verified, as well as dark matter and dark energy are not yet observed in laboratory experiments. Also, GR contains cosmological singularity. These and some other problems give rise to investigate extensions of GR.

In this talk, we present modification of GR extending $R-2\Lambda$ by nonlocal term $\sqrt{R-2\Lambda}\mathcal{F}(\Box)\sqrt{R-2\Lambda}$, where $\mathcal{F}(\Box)$ is an analytic function of the dAlembert operator $\Box$. The choice of $\mathcal{F}(\Box)$ in the analytic form is motivated by existence of analytic expressions with $\Box$ in string field theory and p-adic string theory.

We have found some exact cosmological solutions of the corresponding equations of motion without matter and with $\Lambda \neq 0$. One of these solutions is $a(t) = At^{\frac{2}{3}}e^{\frac{\Lambda}{14}t^2}$, which contains properties similar to an interplay of the dark matter and the dark energy. For this solution we computed some cosmological parameters which are in good agreement with their values in the standard ΛCDM model. This nonlocal gravity model has not the Minkowski space solution. Also, some constraints on function $\mathcal{F}(\Box)$ are obtained.

This is joint work with Branko Dragovich, Alexey Koshelev, Zoran Rakic and Jelena Stankovic, and based on the recent paper arXiv:1906.07560 [gr-qc].

On p-adic and adelic quantum dynamics from locally equivalent DBI Lagrangians

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Motivated by the trans-Planckian problem in the very early stage of cosmological evolution of the Universe, we consider the theoretical role of tachyon scalar field, defined over Archemedean and non-Archimedean spaces.

We consider dynamics of a class of Dirac-Born-Infeld (DBI) type Lagrangians, based on string and D-brane theory, with possible application in inflation theory. Our consideration is done in context of classical and quantum mechanics with tachyon like potentials.

We remind on a formalism for describing dynamics of tachyon fields in spatially homogenous and one-dimensional - classical mechanical limit. Quantization [1] is done in the form of the path integrals on real and p-adic spaces [2], followed by discussion on conditions for their "adelization" [3]. To simplify the equation of motion for the scalar field, canonical transformations are defined and engaged.

We continue with analyzing dynamics of the DBI Lagrangians in four dimensional spaces with FRLW metrics and the form of the obtained quadratic and non-quadratic actions [4].

The corresponding quantum propagators are discussed, as well as possibilities for an adelic generalization and its application in the beginning of inflation [5].

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On Zeta Strings

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In p-adic string theory there is the following effective Lagrangian

$$\mathcal{L}_p = \frac{m^D}{g^2}\frac{p^2}{p-1}\left[-\frac{1}{2}\varphi p^{-\frac{\Box}{2m^2}}\varphi + \frac{1}{p+1}\varphi^{p+1}\right],$$

which describes four point and all higher scattering amplitudes at the tree level of open scalar p-adic tachyons.

Using sum $L = \sum_{n=1}^{+\infty} C_n \mathcal{L}_n$ with a suitable choice of coefficients C_n one obtains a few interesting Lagrangians L containing the Riemann zeta function, which may be related to zeta strings. Applying relevant multiplicative approach, one can also obtain a Lagrangian, similar to the one of the additive approaches. For details, see references [1, 2. 3, 4].

In this talk I will present a few Lagrangians with Riemann zeta function which deserve further investigation.

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Kozyrev wavelets, pseudodifferential operators and a family of zeta functions

Debashis Ghosal

The Kozyrev wavelets on the p-adic space are eigenfunctions of a class of generalised pseudodifferential operators. We show how a family of functions, related to the Riemann zeta function, can be expressed in terms of these operators using the Euler product representation.

A Riemann-Roch theorem on network

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A Riemann-Roch theorem on a connected finite graph was initiated by M. Baker and S. Norine, where connected graph with finite vertices was investigated and unit weight was given on each edge and vertex of the graph. Since a counterpart of the lowest exponents of the complex variable in the Laurent series was proposed as divisor for the Riemann-Roch theorem on graph, its relationships with tropical geometry were highlighted earlier than other complex analytical observations on graphs. On the other hand, M. Baker and F. Shokrieh revealed tight relationships between chip-firing games and potential theory on graphs, by characterizing reduced divisors on graphs as the solution to an energy minimization problem. The objective of this talk is to establish a Riemann-Roch theorem on an edge-weighted infinite graph. We introduce vertex weight assigned by the given weights of adjacent edges other than the units for expression of divisors and assume finiteness of total mass of graph. This is a joint work with A. Atsuji.

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Nonlinear parabolic equations with p-adic spatial variables

Anatolii Kochubei

We consider nonlinear evolution equations for complex-valued functions of a real positive time variable and p-adic spatial variables. In the linear case, there is a well-developed theory of the class of p-adic parabolic equations having both common and different features compared with the classical theory of parabolic equations. In the nonlinear case, we deal with non-Archimedean counterparts of the fractional porous medium equation and the Navier-Stokes equation. Developing, as a tool, an L^q-theory of Vladimirov's p-adic fractional differentiation operator ($1 \leq q < \infty$), we prove m-accretivity of the nonlinear operator corresponding to the equation of the porous medium type, thus obtaining the existence and uniqueness of a mild solution. We also prove the local solvability of the p-adic Navier-Stokes equation.

Analytic Infinite Derivative gravity

Alexey S. Koshelev

University Beira Interior, Portugal

I will explain how theories with analytic functions of derivatives are connected to p-adic strings and will spent the most part of my talk on the good and unrevealed properties of analytic infinite derivative (AID) gravity. In particular, I will discuss questions of avoiding singularities as well as possible observational evidences for such a non-local modification of Einstein's theory. On top of this I will raise several purely mathematical questions which need to be solved in order to push the physics developments further.

p-Adic wavelets in many dimensions

Sergei Kozyrev

Steklov Mathematical Institute, Russia

We discuss wavelet p-adic wavelet bases in many dimensional case. We consider several examples of wavelet bases and discuss relation to coherent states for different groups acting on p-adic spaces and to spectral theory of p-adic pseudodifferential operators.

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Inflation from string field theory

K. Sravan Kumar

In the framework of string field theory (SFT) a setting where the closed string dilaton is coupled to the open string tachyon at the final stage of an unstable brane or brane-anti-brane pair decay is considered. We show that this configuration can lead to viable inflation by means of the dilaton becoming a non-local (infinite-derivative) inflaton. The structure of non-locality leads to interesting inflationary scenarios. We obtain (i) a class of single field inflation with universal attractor predictions at n_s 0.967 and any value of $r < 0.1$, where the tensor to scalar ratio r is solely controlled by the parameters of the SFT; (ii) a new class of two field conformally invariant models with a peculiar quadratic cross-product of scalar fields. We analyze a specific case where a spontaneously broken conformal invariance leads to Starobinsky like inflation plus creating an uplifted potential minimum which accounts to vacuum energy after inflation.

L-packets and a geometric conjecture for $\mathrm{SL}_2(K)$ with K a local function field of characteristic 2

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Let $\mathcal{G} = \mathrm{SL}_2(K)$ with K a local function field of characteristic 2. In [3], the authors studied depth for $\mathcal{G}$ and its inner forms. Continuing the study of the group $\mathcal{G}$ from the point of view of local Langlands correspondence, we review Artin-Schreier theory for the field K, and show that this leads to a parametrization of certain L-packets in the smooth dual of $\mathcal{G}$. We relate this with the geometric conjecture of Aubert-Baum-Plymen-Solleveld [1, 2] (see also [4] for the case of SL_4 over a local field of characteristic zero). Joint work with Roger Plymen.

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Acausal quantum theory for non-Archimedean scalar fields

Maria Luisa Mendoza

The talk aims to show the construction of a family of quantum scalar fields over a $p-$adic spacetime which satisfy $p-$adic analogues of the Gårding–Wightman axioms, this $p-$adic scalar fields satisfy certain $p-$adic Klein-Gordon pseudo-differential equations. We compute explicitly the fundamental solutions of these equations, we also present the second quantization of the solutions of these Klein-Gordon equations which corresponds exactly to the scalar fields introduced. Most of the axioms can be formulated the same way in both, the Archimedean and non-Archimedean frameworks; however, the axioms depending on the ordering of the background field must be reformulated, reflecting the acausality of $p-$adic spacetime. The main conclusion is that there seems to be no obstruction to the existence of a mathematically rigorous quantum field theory (QFT) for free fields in the $p-$adic

framework, based on an acausal spacetime.

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Some p-Adic Properties of the Genetic Code

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The genetic code plays a central role in all living organisms. It connects codons in genes and amino acids in proteins, as well as it determines codons responsible for stop signal in synthesis of proteins. Experimentally, it is well known which concrete codons code any of amino acids, and which codons code stop (terminal) signal. From a mathematical point of view, the genetic code is almost one of $1.5 \cdot 10^{84}$ possible maps from the set of 64 elements (which are codons) onto 21 elements (which are 20 amino acids and 1 stop signal). Theoretically, the main problem is to understand why the genetic code is such as it is.

First theoretical considerations of the genetic code were begun soon after the discovery of the helicoidal structure of DNA by J. Watson and F. Crick (1953). In modeling the genetic code there are many, and rather different, approaches that usually depend on the scientific background of the scientist who is doing it. So, there are physical, mathematical, chemical, biological and some other approaches.

In this talk, we consider p-adic modeling of the standard genetic code and the vertebrate mitochondrial one. To this end, we use 5-adic and 2-adic distance as a mathematical tool to describe closeness (nearness, similarity) between codons as elements of a bioinformation space. Codons which are simultaneously at the smallest 5-adic and 2-adic distance code the same (or similar) amino acid or stop signal. The genetic code can be treated as sequential translation between four genetic languages.

This talk is mainly based on recent publications [1] and [2].

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p-Adic renormalization group phenomena

M.D. Missarov

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Let Q_p^d denotes d-dimensional p-adic space. We consider the four-component field $\psi^*(x) = (\overline{\psi}_1(x), \psi_1(x), \overline{\psi}_2(x)$ over Q_p^d, whose components are generators of the Grassmann algebra. The field model given by the Hamiltonian

$$H(\psi^*; \alpha; r, g) = H_0(\psi^*; \alpha) +$$
$$+ \int \left(r(\overline{\psi}_1(x)\psi_1(x) + \overline{\psi}_2(x)\psi_2(x) + g(\overline{\psi}_1(x)\psi_1(x)\overline{\psi}_2(x)\psi_2(x)\right) dx,$$

whose Gaussian part is invariant w.r.t.the group of scaling transformations $(S_\lambda(\alpha)\varphi)(x) = |\lambda|^{d-\alpha}\varphi(\lambda x)$, where $\lambda \in Q_p$, $\alpha \in R$ is the parameter of this group:

$$H_0(\psi^*; \alpha) = c(\alpha) \int ||x-y||^{-\alpha} \left(\overline{\psi}_1(x)\psi_1(y) + \overline{\psi}_2(x)\psi_2(y)\right) dxdy.$$

Let us consider the discretization of the field ψ^* on the lattice of p-adic fractional vectors T_p^d (hierarchical lattice). Then the discretization of scaling transformation $S_{p^{-1}}(\alpha)$ is given by Kadanoff-Wilson renormalization group transformation on the hierarchical lattice T_p^d. This transformation is computed explicitly as non-lnear transformation in the space of coupling constants (r, g), which is equivalent to quadratic Cremona type transformation of the two-dimensional projective plane. It is possible to describe the global dynamics of this transformation and we observe several new interesting dynamical and symmetry phenomena. Particulary, we can describe the behaviour of all fixed points, its stable invariant curves and new bifurcation phenomena [1]. Some of the discovered renormalization group properties can be generalized to the hierarchical bosonic and Euclidean field models [2]. Exact description of renormalization group flow in the fermionic hierarchical model generates interesting conjectures for the Euclidean models and we discuss some of them.

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Chaos and Phase transitions in p-adic lattice models

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Models of interacting systems have been intensively studied in the last years and new methodologies have been developed in the attempt to understanding their intriguing features. One of the most promising directions is the combination of statistical mechanics tools and methods adopted in dynamical systems. One of such tools is the renormalization group (RG) which has had a profound impact on modern statistical physics. The renormalization method is then applied in statistical mechanics and yielded lots of interesting results. Since such investigations of phase transitions of spin models on hierarchical lattices showed that they make the exact calculation of various physical quantities [1]. One of the most simple hierarchical lattice is a Cayley tree or a Bethe lattice. This lattice is not a realistic lattice, however, investigations of phase transitions of spin models on trees like the Cayley tree show that they make the exact calculation of various physical quantities.

On the other hand, there are many investigations that have been conducted to discuss and debate the question due to the assumption that p-adic numbers provide a more exact and more adequate description of microworld phenomena (see, for example [4, 6]). Consequently, various models in physics described in the language of p-adic analysis and numerous applications of such an analysis to mathematical physics have been studied. These investigations proposed to study new probability models (namely p-adic probability), which cannot be described using ordinary Kolmogorov's probability theory [3]. For complete review of the p-adic mathematical physics we refer to [2].

In [5] we have developed the renormalization group method to p-adic models on Cayley trees. In this work, we are interested in the following question: how is the existence of the phase transition related to chaotic behavior of the associated p-adic dynamical system.The existence of chaos allows us to establish vastness of periodic (with any period) p-adic quasi Gibbs measures for the model. We point out that similar kind of result is not known in the case of real numbers. The main result of this paper allows us to know that how large is the class of p-adic quasi Gibbs measures.

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Ternary Divisive Hierarchical Clustering: Clustering of All that is Exceptional and Anomalous, Counterposed to Commonality, in Big Data Analytics

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Agglomerative hierarchical clustering obtains a binary tree. Here we use a ternary, 3-adic, tree, a divisive hierarchical clustering from the Euclidean metric endowed semantic factor space, mapped from qualitative and quantitative data by Correspondence Analysis, also termed Geometric Data Analysis. This hierarchical clustering can be of linear computational complexity.

Motivation for ternary hierarchical cluster analysis is clustering of all that is exceptional and anomalous, counterposed to commonality. Projections on the factors are used, with positive and negative projections on a factor determining exceptional and anomalous properties, and projections close to the origin being commonality.

We also want to examine this here: hierarchical clustering can be chronological (if time is a variable here) or other adjacency constrained agglomerative clustering. This is to be used for textual narrative, to determine the evolution of emotions. Here the dream texts in [2] are to be analyzed further.

Following [1], here, semantically exceptional data characteristics, in the geometric understanding of the data, that can perhaps also be characterized as anomalous data characteristics, these can well be counterposed to the predominant commonality and typicality in the data that is being analyzed. A very important role in the analytics here is to have the data re-encoded, such as using p-adic data encoding, rather than real-valued data encoding. For text mining, and also for medical and health analytics, the analysis determines a divisive, ternary (i.e. p-adic where p = 3) hierarchical clustering from factor space mapping. The principal applications here are text analysis and medical and health analytics.

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Ultrametric properties of symbolic sequences in theory of quantum chaos.

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Symbolic sequences is a fundamental concept widely used in various fields of natural sciences such as bioinformatics, theory of information and others. In theory of dynamical systems, the symbols are generated by stroboscopic sampling of the multidimensional trajectory. For the case of Hamiltonian system one defines the Poincar section surface. The linearization of the dynamics allows to separate stable and unstable directions of motion and thus define the set of feasible positions at the next crossing of the Poincar section surface. All intersections falling within the same sub-region of the surface are designated by a certain symbol. For the system possessing a chaotic dynamics one can aver existence of the Markov surface partitioning, meaning that each next symbol in the symbolic dynamics is defined only by the previous one. Then the trajectory in phase space can be uniquely restored from the symbolic dynamics. Periodic orbits and, consequently, the cycled symbolic sequences form a skeleton of the dynamical system and can give access by means of semiclassical Gutzwiller trace formula to many of the systems dynamical averages, such as Lyapunov exponents, fractal dimensions, entropy. We show that the set of periodic orbits is an ultrametric space with an ultrametric measure, which definition follows from the physical principles [1,2]. We investigate properties of the obtained set for particular example of a chaotic dynamical system, the bakers map.

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Grossone-based Infinity Computing with Numerical Infinities and Infinitesimals

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In this lecture, a recently developed computational methodology (see a comprehensive description given in [1]) is described. It allows one to execute *numerical* (not symbolic) computations with a variety of infinities and infinitesimals based on the principle *'The part is less than the whole'* applied to all quantities: finite, infinite, and infinitesimal. This methodology does not contradict Cantor, Levi-Civita, and Robinson evolving their ideas in a more applied way and moving from purely symbolic to numerical calculus. The non-contradictoriness of this practical methodology has been studied by several groups of logicians (see, e.g., [2]).

A computational device called the *Infinity Computer* (see EU patent 1728149, RF Patent 2395111, and USA patent 7,860,914) is used to work numerically with infinite and infinitesimal numbers that can be written in a positional system with an infinite base denoted by the symbol ① called *Grossone*. A number written in this numeral system can have several infinite and infinitesimal parts, a finite part can be either present or absent.

On a series of applications it is shown that the new way of counting can be very useful in certain cases. Obtained results and their accuracy are repeatedly compared with answers provided by traditional tools used by mathematicians to work with objects involving infinity.

The *Infinity Calculator* working numerically with the introduced infinities and infinitesimals is shown during the presentation. A lot of an additional information including video lectures, reviews, didactic materials, and more than 50 papers of several authors applying this methodology in their research fields can be found at the dedicated web page http://www.theinfinitycomputer.com

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In this talk, a number of applications showing usefulness of a recent computational methodology allowing one to execute numerical computations with a variety of infinities and infinitesimals is presented. Computations are executed using Euclid's Common Notion no. 5 that is applied to finite, infinite, and infinitesimal numbers. All these numbers are elaborated in a unique framework allowing one to avoid indeterminate forms and divergences giving so the possibility to create new promising computational techniques. For instance, it is shown that sometimes it becomes possible to avoid ill-conditioning of the original problems and substitute iterative procedures leading to approximate results by single computations providing exact answers in one shot (see, e.g., [1]).

During the talk, the main attention will be devoted to summation technics, optimization problems, numerical differentiation, ODEs, measuring infinite sets, and fractals (some of these topics are discussed in [1, 2]).

A lot of an additional information can be found at the dedicated web page `http://www.theinfinitycompu` containing, among other things, papers written by authors working in different research fields. In particular, the ①-based methodology has been applied in: Linear, non-linear, and multi-objective optimization; Numerical differentiation; Ordinary differential equations; Fractals and percolation; Cellular automata; Probability theory; Game theory; Turing machines; Philosophy of mathematics; Set theory; Logic; Mathematical analysis; Riemann zeta function and Dirichlet eta function; Ramanujan summation; Hyperbolic geometry and tiling, etc.

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Calculus theorems for locally uniformly differentiable functions on a non-Archimedean ordered field extension of the real numbers

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Let $\mathcal{N}$ be a non-Archimedean ordered field extension of the real numbers that is Cauchy complete in the topology induced by the order and whose Hahn group is Archimedean. In this talk, I will review the properties of locally uniformly differentiable functions at a point or on a subset of $\mathcal{N}$ and I will formulate and prove a local intermediate value theorem, a local mean value theorem, and a Taylor's theorem for such functions. Then I will extend the concept of locally uniform differentiability to functions from $\mathcal{N}^n$ to $\mathcal{N}^m$ and I will use that to formulate and prove an inverse function theorem for functions from $\mathcal{N}^n$ to $\mathcal{N}^n$ and an implicit function theorem for functions from $\mathcal{N}^n$ to $\mathcal{N}^m$, with $m < n$.

UV properties of loop scattering amplitudes in non-local scalar field theories

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Non-local field theories are interesting from the point of view of quantum gravity. These models allow avoiding ultraviolet divergences in loop scattering amplitudes [1]. Also, non-local lagrangians typically arise in string field theory [2]. From the classical point of view, the non-local lagrangians were intensively studied in the cosmological context, however, the properties of loop corrections were not addressed. In this talk, I present a study of one-loop $2 \to 2$ scattering amplitude in non-local scalar field theory with the action,

$$S = \int d^4x \left(\frac{1}{2}\phi \frac{(\Box - m^2)}{f(\Box)}\phi - \lambda\phi^4 \right) \tag{1}$$

The function $f(\Box)$ should be an analytic entire function without zeros, in order not to produce extra degrees of freedom and ghosts. In this way, the tree level propagator can be written as,

$$P(q) = \frac{-if(q^2)}{q^2 - m^2} \tag{2}$$

If the function $f(\Box)$ goes to zero fast enough the loop integrals become convergent. This makes the scalar theory (1) finite.

The analyticity properties of the function $f(\Box)$ restrict its behavior at (complex) infinity. Namely, for the Laplace theorem, this function must have an essential singularity at infinity and directions of exponentially fast growth at the complex plane. This makes the normally defined Minkowski integral divergent. But this difficulty can be resolved if one defines the scattering amplitudes in another way, through the Euclidian integral [ref sen], in such a way that in the local limit $f(\Box) = 1$ we obtain the familiar result. However, the unitarity within such definition is an open question [sen]. In this work, I show it explicitly for the one-loop diagram. Taking $f(\Box) = e^{-\alpha\Box}$ allows for obtaining analytical result for the amplitude.

Another generic result regarding properties of non-local amplitudes can be obtained when applying dispersion relations technic. Assume that the optical theorem is satisfied in (1) and that the function $f(\Box)$ is chosen in such a way that the forward scattering amplitude $\mathcal{A}(s, t = 0)$ is an analytic function on the whole complex plane except two branch cuts along the real axis (corresponding to the decay of the scalar). Than, one can write,

$$\mathcal{A}(\mu^2) = \frac{1}{2\pi i}\oint_\Gamma ds\, \frac{\mathcal{A}(s)}{(s-\mu^2)} = \lambda^2 \int_{4m^2}^{\infty} ds \sqrt{s(4m^2 - s)} \left(\frac{1}{(s-\mu^2)} + \frac{1}{(4m^2 - s - \mu^2)} \right) \tag{3}$$

under an additional assumption that the integral over an infinite circle is zero. Note that the integral in the right hand side does not contain the function $f(\Box)$. This means that there is no way to make this integral convergent. Thus, we can conclude that in a UV finite non-local theory we will obtain poles in the scattering amplitude, independently of the concrete form of the function.

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p-Adic noncommutative calculus

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On linear isometries on strongly regular non-Archimedean Köthe spaces

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We study linear isometries on strongly regular Köthe spaces. The most known and important examples of nuclear strongly regular Köthe spaces are the generalized power series spaces $D_f(a, r)$.

By a *Köthe space* we mean an infinite-dimensional Fréchet space with a Schauder basis and with a continuous norm. An infinite matrix $A = (a_n^k)$ of positive real numbers is a *Köthe matrix* if $a_n^k \leq a_n^{k+1}$ for all $k, n \in \mathbb{N}$. The space $K(A) = \{(\alpha_n) \subset \mathbb{K} : \lim_n |\alpha_n| a_n^k = 0 \text{ for every } k \in \mathbb{N}\}$ with the base (p_k) of norms, where $p_k((\alpha_n)) = \max_n |\alpha_n| a_n^k, k \in \mathbb{N}$, is a Köthe space. The sequence (e_j), where $e_j = (\delta_{j,n})$, is an unconditional Schauder basis in $K(A)$. Any Köthe space is isomorphic to the space $K(A)$ for some Köthe matrix A.

For non-empty subsets $X, Y \subset \mathbb{N}$ we write $X < Y$ if $x < y$ for all $x \in X, y \in Y$. A Köthe matrix $A = (a_n^k)$ is *strongly regular* if there exists a partition (N_s) of $\mathbb{N}$ onto non-empty finite subsets such that (1) $a_i^k = a_j^k$ for all $i, j \in N_s, s \in \mathbb{N}$; (2) $a_i^{k+1}/a_i^k < a_j^{k+1}/a_j^k$ for all $i \in N_s, j \in N_{s+1}, s, k \in \mathbb{N}$; (3) $N_s < N_{s+1}$ for every $s \in \mathbb{N}$. We say that a Köthe space $K(A)$ is *strongly regular* if A is strongly regular.

We study when two strongly regular Köthe spaces $K(A)$ and $K(B)$ are isometrically isomorphic.

Let $A = (a_n^k)$ and $B = (b_n^k)$ be strongly regular Köthe matrices. The Köthe spaces $K(A)$ and $K(B)$ are isometrically isomorphic if and only if there exists a sequence $(\psi_n) \subset \mathbb{K}^*$ such that $a_n^k = |\psi_n| b_n^k$ for all $k, n \in \mathbb{N}$. In this case the linear map $P : K(A) \to K(B), x = (x_n) \to Px = (\psi_n x_n)$ is an isometric isomorphism.

We determine all linear isometries on a strongly regular Köthe space $K(A)$.

Let $A = (a_n^k)$ be a strongly regular Köthe matrix. Let (N_s) be a partition of $\mathbb{N}$ associated with A. A continuous linear map $T : K(A) \to K(A)$ with $Te_j = \sum_{i=1}^{\infty} t_{i,j} e_i, j \in \mathbb{N}$, is an isometry if and only if $|t_{i,j}| \leq \inf_k a_j^k / a_i^k$ for all $i, j \in \mathbb{N}$ and $|\det[t_{i,j}]_{i,j \in N_s}| = 1$ for every $s \in \mathbb{N}$.

We also prove that any linear isometry on a nuclear strongly regular Köthe space $K(A)$ is surjective.

Reaction-diffusion Equations on Complex Networks and Turing Patterns, via p-Adic Analysis

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The central purpose of the talk is to show that p-adic analysis is the natural tool to study, in a rigorous mathematical way, reaction-diffusion systems on networks and the corresponding Turing patterns. By embedding the graph attached to the system into the field of p-adic numbers, we construct a family of continuous p-adic versions of the original system. In this way, we can study the original system and to obtain a new p-adic continuous version of it, which corresponds to a 'mean-field approximation' of the original system. The existence and uniqueness of the Cauchy problems for all these systems are established. We show that Turing criteria remains essentially the same as in the classical case. However, the properties of the emergent patterns are very different. The classical Turing patterns consisting of alternating domains do not occur in the network case, instead of this, several domains (clusters) occur. Multistability, that is, coexistence of a number of different patterns for the same parameters values occurred.